On the Smallness of the Cosmological Constant in SUGRA Models Inspired by Degenerate Vacua

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Abstract. In the no–scale supergravity global symmetries protect local supersymmetry and a zero value for the cosmological constant. The breakdown of these symmetries, which ensures the vanishing of the vacuum energy density, results in a set of degenerate vacua with broken and unbroken supersymmetry leading to the natural realisation of the multiple point principle (MPP). In the MPP inspired SUGRA models the cosmological constant is naturally tiny.

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INTRODUCTION

Recent observations indicate that 70%-73% of the energy density of the Universe exists in the form of dark energy. This tiny vacuum energy density (the cosmological constant) $\Lambda \sim 10^{-123} M_{Pl}^4 \sim 10^{-55} M_Z^4$ is responsible for the accelerated expansion of the Universe. In the standard model (SM) the cosmological constant is expected to be many orders of magnitude larger than the observed vacuum energy density. Indeed, much larger contributions must come from the QCD condensates and electroweak symmetry breaking, while the contribution of zero–modes should push the total vacuum energy density up to $\sim M_{Pl}^4$. An exact global supersymmetry (SUSY) ensures zero value for the vacuum energy density. However the breakdown of SUSY induces a huge and positive contribution to the cosmological constant of order M_S^4, where SUSY breaking scale $M_S \gg 100\,\mathrm{GeV}$.

MPP INSPIRED SUGRA MODELS

In general the vacuum energy density in $(N=1)$ supergravity (SUGRA) models is huge and negative $\Lambda \sim -m_{3/2}^2 M_{Pl}^2$, where $m_{3/2}$ is a gravitino mass. The situation changes dramatically in no-scale supergravity where the invariance of the Lagrangian under imaginary translations and dilatations results in the vanishing of the vacuum energy density. Unfortunately these global symmetries also protect supersymmetry which has to be broken in any phenomenologically acceptable theory. The breakdown of dilatation invariance does not necessarily result in a non–zero vacuum energy density [1]–[4]. Let

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us consider a SUGRA model that involves two hidden sector fields (T and z) and a set of chiral supermultiplets φ_σ in the observable sector, which transform differently under the imaginary translations ($T \to T + i\beta$, $\varphi_\sigma \to \varphi_\sigma$, $z \to z$) and dilatations ($T \to \alpha^2 T$, $z \to \alpha z$, $\varphi_\sigma \to \alpha\,\varphi_\sigma$). In the considered SUGRA model the superpotential W and K$\ddot{a}$hler potential K can be written in the following form [1]-[3]:

$$
\begin{gathered}
W(z,\varphi_\alpha) = \kappa\left(z^3 + \mu_0 z^2 + \sum_{n=4}^{\infty} c_n z^n\right) + \sum_{\sigma,\beta,\gamma} \frac{1}{6} Y_{\sigma\beta\gamma}\varphi_\sigma\varphi_\beta\varphi_\gamma\,, \\
K = -3\ln\left[T + \overline{T} - |z|^2 - \sum_\sigma \zeta_\sigma |\varphi_\sigma|^2\right] + \sum_{\sigma,\lambda}\left(\frac{\eta_{\sigma\lambda}}{2}\varphi_\sigma\varphi_\lambda + h.c.\right) + \sum_\sigma \xi_\sigma |\varphi_\sigma|^2.
\end{gathered}
\tag{1}
$$

Here we use standard supergravity mass units: $\frac{M_{Pl}}{\sqrt{8\pi}} = 1$. In Eq. (1) we include a bilinear mass term for the superfield z and higher order terms $c_n z^n$ in the superpotential that spoil dilatation invariance. We also allow the breakdown of dilatation invariance in the K$\ddot{a}$hler potential of the observable sector which is caused by a set of terms $\xi_\sigma|\varphi_\sigma|$ and $\eta_{\alpha\beta}\varphi_\alpha\varphi_\beta$. At the same time we do not allow the breakdown of dilatation invariance in the superpotential of the observable sector to avoid the appearance of potentially dangerous terms which lead, for instance, to the so–called μ–problem and in the K$\ddot{a}$hler potential of the hidden sector.

In the considered SUGRA model the scalar potential of the hidden sector is positive definite

$$
V(T,z) = \frac{1}{3(T+\overline{T}-|z|^2)^2}\left|\frac{\partial W(z)}{\partial z}\right|^2, \tag{2}
$$

so that the vacuum energy density vanishes near its global minima. In the simplest case when $c_n = 0$, $V(T,z)$ has two minima at $z = 0$ and $z = -\frac{2\mu_0}{3}$. In the first vacuum, where $z = -\frac{2\mu_0}{3}$, local SUSY is broken so that the gravitino becomes massive

$$
m_{3/2} = \left\langle \frac{W(z)}{(T+\overline{T}-|z|^2)^{3/2}} \right\rangle = \frac{4\kappa\mu_0^3}{27\left\langle\left(T+\overline{T}-\frac{4\mu_0^2}{9}\right)^{3/2}\right\rangle} \tag{3}
$$

and all scalar particles get non–zero masses $m_\sigma \sim \frac{m_{3/2}\xi_\sigma}{\zeta_\sigma}$. In the second minimum, with $z = 0$, the superpotential of the hidden sector vanishes and local SUSY remains intact, so that the low–energy limit of this theory is described by a pure SUSY model in flat Minkowski space. If the high order terms $c_n z^n$ are present in Eq. (1), the scalar potential of the hidden sector may have many degenerate vacua with broken and unbroken supersymmetry in which the vacuum energy density vanishes.

Thus the considered breakdown of dilatation invariance leads to a natural realisation of the multiple point principle (MPP). The MPP postulates the existence of many phases with the same energy density which are allowed by a given theory [5]-[6]. In SUGRA models of the above type there is a vacuum in which the low–energy limit of the considered theory is described by a pure supersymmetric model in flat Minkowski space. According to the MPP this vacuum and the physical one in which we live must

be degenerate. Such a second vacuum is only realised if the SUGRA scalar potential has a minimum where $m_{3/2}=0$ which normally requires an extra fine-tuning [7]. In the SUGRA model considered above the MPP conditions are fulfilled automatically without any extra fine-tuning at the tree–level.

THE VALUE OF THE COSMOLOGICAL CONSTANT

Because the vacuum energy density of supersymmetric states in flat Minkowski space is zero and all vacua in the MPP inspired SUGRA models are degenerate, the cosmological constant problem is solved to first approximation by our assumption. However non–perturbative effects in the observable sector can give rise to the breakdown of SUSY in the second vacuum (phase). If SUSY breaking takes place in the second vacuum, it is caused by the strong interactions. When the gauge couplings at high energies are identical in both vacua the scale Λ_{SQCD}, where the QCD interactions become strong in the second vacuum, is given by

$$\Lambda_{SQCD}=M_S\exp\left[\frac{2\pi}{b_3\alpha_3^{(2)}(M_S)}\right], \qquad \frac{1}{\alpha_3^{(2)}(M_S)}=\frac{1}{\alpha_3^{(1)}(M_Z)}-\frac{\tilde{b}_3}{4\pi}\ln\frac{M_S^2}{M_Z^2}, \qquad (4)$$

where M_S is the SUSY breaking scale in the physical vacuum. In Eq.(4) $\alpha_3^{(1)}$ and $\alpha_3^{(2)}$ are the values of the strong gauge couplings in the physical and second minima of the SUGRA potential, while $\tilde{b}_3=-7$ and $b_3=-3$ are the one–loop beta functions of the SM and MSSM. At the scale Λ_{SQCD} the t–quark Yukawa coupling in the MSSM is of the same order of magnitude as the strong gauge coupling. The large Yukawa coupling of the top quark may result in the formation of a quark condensate that breaks supersymmetry inducing a non–zero positive value for the cosmological constant $\Lambda\simeq\Lambda_{SQCD}^4$. The MPP philosophy then requires that the physical phase in which local supersymmetry is broken in the hidden sector has the same energy density as a second phase where non–perturbative supersymmetry breakdown takes place in the observable sector.

In Fig. 1 the dependence of Λ_{SQCD} on the SUSY breaking scale M_S is examined. Because $\tilde{b}_3<b_3$ the QCD gauge coupling below M_S is larger in the physical minimum than in the second one. Therefore the value of Λ_{SQCD} is much lower than the QCD scale in the Standard Model and diminishes with increasing M_S. When the SUSY breaking scale in our vacuum is of the order of 1 TeV, we obtain $\Lambda_{SQCD}^4=10^{-104}M_{Pl}\simeq$ 100 eV which is much smaller than an electroweak scale contribution in our vacuum $v^4\simeq10^{-62}M_{Pl}$. From the rough estimate $\Lambda\simeq\Lambda_{SQCD}^4$ of the energy density, it can be easily seen that the measured value of the cosmological constant is reproduced when $\Lambda_{SQCD}=10^{-31}M_{Pl}\simeq10^{-3}$ eV [1], [7] which is attained for $M_S=10^3-10^4\,$TeV. However the consequent large splitting within SUSY multiplets would spoil gauge coupling unification and reintroduce the hierarchy problem, which would make the stabilisation of the electroweak scale rather problematic.

A model consistent with electroweak symmetry breaking and cosmological observations can be constructed, if the MSSM particle content is supplemented by an additional pair of $5+\overline{5}$ multiplets. In the physical vacuum these extra particles would

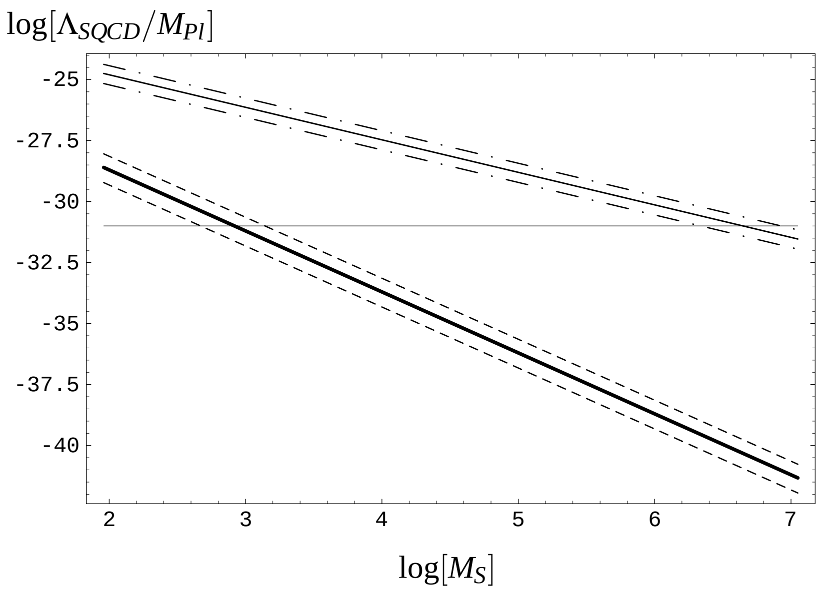

FIGURE 1. The value of $\log\left[\Lambda_{SQCD}/M_{Pl}\right]$ versus $\log M_S$. The thin and thick solid lines correspond to the pure MSSM and the MSSM with an extra pair of $5+5$ multiplets. The dashed and dash–dotted lines represent the uncertainty in $\alpha_3(M_Z)$, i.e. $\alpha_3(M_Z) = 0.112 - 0.124$. The horizontal line corresponds to the observed value of $\Lambda^{1/4}$. The SUSY breaking scale M_S is given in GeV.

gain masses around the supersymmetry breaking scale due to the presence of the bilinear terms $\left[\eta(5\cdot\overline{5}) + h.c.\right]$ in the K$\ddot{a}$hler potential [1]. Near the second minimum of the SUGRA scalar potential the new particles would be massless, since $m_{3/2} = 0$. Therefore they give a considerable contribution to the β functions ($b_3 = -2$), reducing Λ_{SQCD} further. In this case the observed value of the cosmological constant can be reproduced even for $M_S \simeq 1\,\text{TeV}$ (see Fig. 1) [1], [7].

CONCLUSIONS

We have argued that the breakdown of global symmetries in no-scale supergravity can lead to a set of degenerate vacua with broken and unbroken local supersymmetry (first and second phases) so that the MPP conditions are satisfied without any extra fine-tuning. In the MPP inspired SUGRA models supersymmetry in the second phase may be broken dynamically in the observable sector inducing a tiny and positive value of the cosmological constant which can be assigned, by virtue of the MPP to all other phases.

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